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A REAL-TIME ALGORITHM FOR ATMOSPHERIC CORRECTIONS OF AIRBORNE REMOTE OPTICAL MEASUREMENTS ABOVE THE OCEAN*

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ABSTRACT

The approximate solution to the equation of transfer for optical radiation in the atmosphere above the sea is obtained. For a zeroth approximation derivation of the equation of transfer, a bi-transport representation of the light scattering phase function was used. Already in this approximation an equation for the transmittance of the atmosphere is obtained within an accuracy of about 5%. The next approximation leads to expressions for radiance factors of the sky and ocean-atmospheric-layer system, the latter of which, in the case of an homogeneous atmosphere and not very oblique sensing and solar angles, transforms at the upper boundary of the atmosphere into the well-known formulae used for processing optical satellite data. The results of this study can be used to develop algorithms for processing airborne and shipborne optical information.

1.0 INTRODUCTION

A significant portion of the signal detected by an airborne optical sensor is due to atmospheric scattering. So in order to correctly process optical data collected by aircraft sensors, it is necessary to have fast, reliable algorithms to correct for atmospheric effects. In spite of the fact that there are several well-established algorithms for processing optical data measured from satellite (Gordon and Wang, 1994; Viollier, Tanre and Deschamps, 1980) there is a shortage of similar airborne algorithms. The reason for this possibly lies in the greater complexity involved. The airborne algorithms utilize not only upward but also downward radiances, and consequently need twice as many input parameters in comparison with the satellite measurements.

In this presentation a derivation of an atmospheric corrections algorithm for airborne optical measurements of the sea surface radiance is given and resulting analytical equations are explained. The equations are obtained as an approximate solution to the equation of transfer in a cloudless scattering atmosphere above the sea. They give values of the ocean-atmospheric and sky radiance coefficients as a function of total atmospheric optical thickness, atmospheric optical thickness below the aircraft, total scattering phase-function, surface reflection parameters as well as viewing angles. The simplicity of these analytical equations makes it possible to convert them into a real-time algorithm for airborne optical atmospheric corrections. The algorithm is suitable for atmospheric corrections of aircraft-measured radiances in an ocean-atmospheric system, and also can be used as an alternative algorithm for atmospheric corrections of satellite data under conditions associated with a cloudless marine atmosphere.

A comparison of results given by the proposed approach with previous algorithms (Gordon and Wang, 1994; and Viollier, Tanre and Deschamps, 1980) shows that its accuracy lies within the

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range of precision of these algorithms for satellite measurements. It is also shown that in the case of aircraft measurements, where the above-mentioned algorithms are not applicable, the range of precision of the proposed method is the same or better.

Simple, approximate solutions to the radiation transfer equation which describe the light fields in the atmosphere over the sea with a sufficient degree of accuracy play an important role in the creation of algorithms for analysis of remotely measured radiance spectra of the ocean-atmospheric system. The radiance spectra of upwelling optical radiation at the upper boundary of atmosphere can be described by simple semi-empirical models (Deschamps, Herman and Tanre, 1983; Viollier, Tanre and Deschamps, 1980) proposed for the processing of data measured by the CZCS scanner which was in operation on the "NIMBUS-7" satellite. Up to that point no similar algorithm suitable for analyzing aircraft measurements of the ocean-atmospheric radiance had been proposed.

In this study the radiance of light in the atmosphere above the ocean is calculated. The formulae obtained here have a sufficiently simple form for use in real-time algorithms for processing shipborne and airborne measurements of atmospheric radiance. With additional limitations imposed on the magnitudes of the optical thickness of the atmosphere, the zenith angles of the sun and the scanning angle, the results acquire an extremely simple form. It is interesting to mention that the equation for the radiance reflection coefficient of the ocean-atmospheric system at the satellite level turns into the basic formula from the model given in earlier work by Viollier, Tanre and Deschamps (1980).

2.0 DERIVATION OF THE BASIC EQUATIONS

Here interest is confined to the visible part of the light spectrum where all absorbing agents but ozone can be neglected. The ozone component is concentrated mainly in the upper atmospheric layer and can be considered separately. So the part of the atmosphere below the ozone layer can be reasonably regarded as a scattering layer only. For this reason the starting point is the following equation of transfer for optical radiation in a scattering and non-absorbing atmosphere:

$$\left(\mu\frac{\partial}{\partial\tau}+1\right)L(\tau,\mu,\varphi)=\frac{1}{4\pi}\int_{\Omega}L(\tau,\mu',\varphi')\,p(\cos\chi)\,d\Omega', \tag{1}$$

where $L(\tau,\mu,\varphi)$ is the spectral radiance of the light propagating in the direction determined by the polar angle $\cos^{-1}\mu$ and azimuthal angle φ at the optical depth $\tau=\int_0^z s(z')\,dz'$, $s(z)$ is the coefficient of light scattering at an altitude z from the ocean surface, $p(\cos\chi)$ is the phase function of light scattering by air with hydrosols at angle

$$\chi=\cos^{-1}\left\{\mu\mu'+\sqrt{\left(1-\mu^2\right)\left(1-\mu'^2\right)}\cos\left(\varphi-\varphi'\right)\right\}, \tag{2}$$

Ω is the full solid angle and $d\Omega=d\varphi\,d\mu$ is a solid angle differential element. In Eqn. (1) it was assumed that the only absorbing substance in the atmosphere is the ozone contained in the narrow layer situated above the layer of scattering atmosphere (Viollier, Tanre and Deschamps, 1980).

Let Ω_+ be the hemisphere of the full solid angle Ω for which $0\le\mu\le1$, $0\le\phi\le\pi$. Keeping in mind that $L(\tau,\mu,\phi\pm2\pi)=L(\tau,\mu,\phi)$, the following designations are introduced:

$$L_1(\tau,\mu,\phi)=L(\tau,\mu,\phi),\quad L_2(\tau,\mu,\phi)=L(\tau,-\mu,\phi+\pi),\quad (\mu,\phi)\in\Omega_+, \tag{3}$$

where L_1 is the upwelling radiance of the ocean light field, while L_2 is the radiance of the downwelling light.

The upwelling (E_1) and downwelling (E_2) irradiances are defined as:

$$E_1(\tau) = \int_{\Omega_+} L_1(\tau,\mu,\phi)\,\mu\, d\Omega, \quad E_2(\tau) = \int_{\Omega_+} L_2(\tau,\mu,\phi)\,\mu\, d\Omega. \tag{4}$$

The following boundary conditions for Eqn. (1) are specified:

a) at the upper boundary of the atmosphere below the ozone layer:

$$L_2(\tau^*,\mu,\phi) = \pi F\,\delta(\mu-\mu_s), \quad (\mu,\phi)\in\Omega_+, \tag{5}$$

where $\tau^* = \int_0^\infty s(z)\,dz$ is the total optical thickness for atmospheric scattering, $F = S\exp(-\tau_{oz}/\mu_s)$, πS is the solar constant, τ_{oz} is the total optical thickness of the absorbing ozone layer, $\mu_s = \cos z_s$, where z_s is the sun zenith angle and the azimuthal angle of the sun $\phi_s = 0$, and $\delta(\mu)$ is the Dirac delta function.

b) at the bottom of the atmosphere just above the surface of the ocean $(\tau = +0)$:

$$L_1(0,\mu,\phi) = \frac{A}{\pi}E_2(0) + R_F(\mu)\,L_2(0,\mu,\phi-\pi), \; (\mu,\phi)\in\Omega_+, \tag{6}$$

where

$$R_F(\mu) = \left(\frac{\mu - \sqrt{n_w^2+\mu^2-1}}{\mu + \sqrt{n_w^2+\mu^2-1}}\right)^2 \frac{\left(1-\mu^2\right)^2 + \mu^2(n_w^2+\mu^2-1)}{\left[\left(1-\mu^2\right) + \mu\sqrt{n_w^2+\mu^2-1}\right]^2} \tag{7}$$

is the Fresnel reflection coefficient for unpolarized light falling on the ocean surface at the angle $\theta = \cos^{-1}\mu$ to the normal, n_w is the index of refraction of light by sea water, $A = E_1^D(0)/E_2(0) = A_{fg} + r'$ is the Lambert part of the albedo of the sea, A_{fg} is the effective albedo of the foam and of specks of light scattered to the sides from the direction of the light reflected by the ocean surface, $r' = (1-f_f)r$, f_f is the portion of the sea surface covered with foam and r is the sea radiance reflection coefficient.

The phase function of light scattering by air with hydrosols $p(\cos\chi)$ is strongly elongated in the forward direction. In order to find the zeroth approximation to the solution of Eqn. (1) the following bi-transport representation (Davison, 1957) of the phase function can be used:

$$\begin{aligned} p(\cos\chi) &= 2\left[(1-B)\,\delta(1-\cos\chi) + B\,\delta(1+\cos\chi)\right] = \\ &4\pi\left[(1-B)\,\delta(\phi-\phi')\,\delta(\mu-\mu') + B\,\delta(\pi+\phi-\phi')\,\delta(\mu+\mu')\right], \end{aligned} \tag{8}$$

where $B = 0.5\int_{-1}^{0} p(\eta)\,d\eta$, $(\eta = \cos\chi)$ is the probability of backscattering.

Taking into account Eqns. (3) and (8), Eqn.(1) becomes a system of equations for determining L_1 and L_2 in the zeroth approximation:

$$\left.\begin{aligned} \left(\mu\frac{\partial}{\partial\tau} + B\right)L_1^0(\tau,\mu,\phi) - B\,L_2^0(\tau,\mu,\phi) = 0, \\ -B\,L_1^0(\tau,\mu,\phi) + \left(-\mu\frac{\partial}{\partial\tau} + B\right)L_2^0(\tau,\mu,\phi) = 0. \end{aligned}\right\} \tag{9}$$

Since the determinant of the system of Eqns. (9) is confluent (*i. e.* both eigenvalues coincide), the solution is assumed to be of the form

$$L_1^0(\tau,\mu,\phi) = A_1(\mu,\phi) - C(\mu,\phi)\,\tau, \;\; L_2^0(\tau,\mu,\phi) = A_2(\mu,\phi) + D(\mu,\phi)\,\tau. \tag{10}$$

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The solvability condition of the system of Eqns. (9) gives the following two relationships between the functional coefficients A_1, A_2, C and D:

$$A_2 = A_1 - \mu C / B, \quad D = -C. \tag{11}$$

Applying the boundary condition given by Eqn. (5) to the solutions (10) together with conditions (11) gives:

$$L_1^0(\tau,\mu,\phi) = \pi F \delta(\phi)\delta(\mu-\mu_s) + \left[\mu + B(\tau^* - \tau)\right] C(\mu,\phi), \tag{12}$$

$$L_2^0(\tau,\mu,\phi) = \pi F \delta(\phi)\delta(\mu-\mu_s) + B(\tau^* - \tau) C(\mu,\phi). \tag{13}$$

By inserting Eqn. (10) into the second boundary condition given by Eqn. (6), and taking into account that the right-hand side of L_2^0 can be expressed with the help of a constant F:

$$L_2^0(\tau,\mu,\phi) = \pi F \exp(-\tau^*/\mu_s)\delta(\phi)\delta(\mu-\mu_s), \tag{14}$$

the following integral equation for determining the functional coefficient $C(\mu,\phi)$ is obtained

$$\begin{aligned}(\mu + B\tau^*)\, C(\mu,\phi) &= \pi F \delta(\mu-\mu_s)\left[g_s\, \delta(\phi-\pi) - \delta(\phi)\right] \\ &+ \frac{AB\tau^*}{\pi}\int_0^{2\pi} d\phi' \int_0^1 C(\mu',\phi')\mu'\, d\mu' + AF\mu_s,\end{aligned} \tag{15}$$

where

$$g_s = g(\tau^*,\mu_s), \quad g = g(\tau,\mu) \equiv R_F(\mu)\exp(-\tau/\mu). \tag{16}$$

Because Eqn. (15) involves both smooth and singular (delta-shaped) functions it is convenient for further calculations to also represent an auxiliary function $C(\mu,\phi)$ as a sum of singular and smooth terms:

$$C(\mu,\phi) = \left[C_1\delta(\phi) + C_2\,\delta(\phi-\pi)\right]\delta(\mu-\mu_s) + AF\,D(\mu), \tag{17}$$

where $D(\mu)$ is a newly introduced smooth function and C_1 and C_2 are coefficients which determine the singular part of the function $C(\mu,\phi)$. In order to derive an equation for $D(\mu)$ Eqn. (17) is inserted into Eqn. (15). After simple algebraic manipulations, expressions are obtained for the coefficients C_1 and C_2:

$$C_1 = -\frac{\pi F}{\mu_s + B\tau^*}, \quad C_2 = \frac{\pi F g}{\mu_s + B\tau^*}, \tag{18}$$

and the integral equation for determining the smooth function $D(\mu)$

$$(\mu + B\tau^*)D(\mu) = \mu_s\left[\frac{1+gBm}{1+Bm} + 2ABm\int_0^1 D(\mu')\mu'\,d\mu'\right], \tag{19}$$

where $m = \tau^*/\mu_s$ denotes the atmospheric mass. Because the right side of Eqn. (19) does not depend on μ its solution is assumed to have the following form:

$$D(\mu) = \frac{C_3}{\mu + B\tau^*}, \quad 0 \le \mu \le 1. \tag{20}$$

Inserting Eqn. (20) into Eqn. (17):

$$C_3 = \frac{\mu_s(1+g\,Bm)}{(1-A\Psi)(1+Bm)}, \tag{21}$$

where

$$\Psi = 2B\tau^*\left[1 - B\tau\ln\left(1+\frac{1}{B\tau^*}\right)\right] \cong \begin{cases} 2B\tau^*, & B\tau^* << 1, \\ 1 - \dfrac{2}{3B\tau^*}, & B\tau^* >> 1. \end{cases} \tag{22}$$

Thus, the solution of the system of Eqns. (9) for irradiances takes the form

$$L_1^0(\tau,\mu,\varphi) = \pi F\left[\frac{B\tau\,\delta(\varphi) + g\left[1-B(\tau^*-\tau)\right]\delta(\varphi-\pi)}{\mu_s(1+Bm)}\right]\delta(\mu-\mu_s)$$
$$+\ \ F A\,\frac{\mu_s T_s\left[\mu + B(\tau^*-\tau)\right]}{\mu + B\tau^*}, \qquad (\mu,\varphi)\in\Omega_+, \tag{23}$$

$$L_2^0(\tau,\mu,\varphi) = \pi F\left[\frac{(\mu_s+B\tau)\delta(\varphi) + g_s B(\tau^*-\tau)\,\delta(\varphi-\pi)}{\mu_s(1+Bm)}\right]\times$$
$$\delta(\mu-\mu_s)\ +\ F A\,\frac{\mu_s T_s B(\tau^*-\tau)}{\mu+B\tau^*}, \qquad (\mu,\varphi)\in\Omega_+, \tag{24}$$

where

$$T_s = \left(1 + g_s Bm\right)/\left[(1-A\Psi)(1+Bm)\right]. \tag{25}$$

Here T_s stands for the transmittance of the entire atmospheric layer. Even in this approximation the results for irradiances can be very useful. When $g_s = A = 0$, Eqn. (25) for transmittance coincides with the appropriate results of studies by Deschamps Herman and Tanre (1983).

Using Eqns. (23) and (24) for the irradiances a number of equations useful for the practical purposes of restoration of atmospheric parameters from the ship or aircraft can be derived. By applying the definitions given by Eqn. (4), the irradiances of the horizontal surface from above $(i=2)$ and from below $(i=1)$ become:

$$E_1^0(\tau) = \pi F\mu_s\left\{g_s + AT_s + \frac{\tau}{\tau^*}\left[\frac{(1-g_s)Bm}{1+Bm} - AT_s\Psi\right]\right\}, \tag{26}$$

$$E_2^0(\tau) = \pi F\mu_s T, \tag{27}$$

where

$$T = T_s + \frac{\tau}{\tau^*}\left(1-T_s\right) \tag{28}$$

is the transmittance of the atmospheric layer which lies above the horizontal plane $\tau = const$.

In addition to utilizing measurements of horizontal surfaces measurements of irradiances from planes vertical to the sea surface can be used. For example, formulae for the irradiance of the area perpendicular to the plane of the solar vertical, from the direction of the sun $(E_\perp^s)$ and in the opposite direction $(E_\perp^D)$ ($E_\perp^s \equiv E_{21}^0,\ E_\perp^D \equiv E_{12}^0$) can be derived:

$$E_{ik}^0 = \int_{-\pi/2}^{\pi/2}\cos\varphi\,d\varphi\int_0^1 L_i^0\sqrt{1-\mu^2}\,d\mu\ -\ \int_{\pi/2}^{3\pi/2}\cos\varphi\,d\varphi\int_0^1 L_k^0\sqrt{1-\mu^2}\,d\mu. \tag{29}$$

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Taking corresponding integrals:

$$E_{\perp}^{s}(\tau)=\pi F\left\{(1-\mu_s^2)\frac{1+g_s+Bm\left[g_s+(1-g_s)\tau^*/\tau\right]}{1+Bm}+A\,\mu_s\,T_s\,T_\omega\right\}, \tag{30}$$

$$E_{\perp}^{D}(\tau)=\pi F\left\{(1-\mu_s^2)\frac{Bm\left[g_s+(1-g_s)\tau^*/\tau\right]}{1+Bm}+A\,\mu_s\,T_s\,T_\omega\right\}, \tag{31}$$

where

$$T_\omega=1+\omega\left(1-\frac{2\tau}{\tau^*}\right),\quad \omega=\frac{2B\tau^*}{\pi}\int_0^1\frac{\sqrt{1-\mu^2}\;d\mu}{B\tau^*+\mu}, \tag{32}$$

$$\omega=\left(B\tau^*\right)^2-\frac{2B\tau^*}{\pi}\left[1+2\sqrt{\left|1-\left(B\tau^*\right)^2\right|}\,W(B\tau^*)\right], \tag{33}$$

$$W(B\tau^*)=\begin{cases}\dfrac{1}{2}\ln(2B\tau^*)-\ln\left[\sqrt{1+B\tau^*}+\sqrt{1-B\tau^*}\right], & B\tau^*\le 1,\\ \tan^{-1}\sqrt{\dfrac{B\tau^*-1}{B\tau^*+1}}, & B\tau^*>1.\end{cases} \tag{34}$$

At the level corresponding to the ocean boundary $(\tau=0)$:

$$E_{\perp}^{s}=\pi F\mu_s\left[\left(\frac{1}{\mu_s}-\mu_s\right)\left(g_s+\frac{1}{1+Bm}\right)+AT_s(1+\omega)\right], \tag{35}$$

$$E_{\perp}^{D}=\pi F\mu_s\left[\left(\frac{1}{\mu_s}-\mu_s\right)\frac{Bm}{1+Bm}+AT_s(1+\omega)\right]. \tag{36}$$

Note that Eqns. (25), (35) and (36) can be used as the basis of an algorithm to measure the atmospheric optical thickness τ^*, the backscattering probability B and the ocean albedo A from the ship.

Let us calculate the radiances of the upwelling and downwelling radiation in the next approximation by expressing the right side of Eqn. (1) and both boundary conditions (5) and (6) via solutions given by Eqns. (23) and (24):

$$\begin{aligned}L_1^1(\tau,\mu,\varphi)&=\left[AF\mu_sT_s+R_F(\mu)\,L_2^1(0,\mu,\varphi-\pi)\right]\exp(-\tau/\mu)+\\&+\frac{\exp(-\tau/\mu)}{4\pi\mu}\int_0^{\tau}d\tau'\exp(\tau'/\mu)\int_{\Omega_+}d\Omega'\left[p(\cos\chi)L_1^0(\tau',\mu',\varphi')+p(-\cos\chi)L_2^0(\tau',\mu',\varphi')\right],\end{aligned} \tag{37}$$

$$\begin{aligned}L_2^1(\tau,\mu,\varphi)&=\pi F\delta(\varphi)\delta(\mu-\mu_s)\exp\left[-(\tau^*-\tau)/\mu_s\right]+\\&+\frac{\exp(\tau/\mu)}{4\pi\mu}\int_{\tau}^{\tau^*}d\tau'\exp(-\tau'/\mu)\int_{\Omega_+}d\Omega'\left[p(-\cos\chi)L_1^0(\tau',\mu',\varphi')+p(\cos\chi)L_2^0(\tau',\mu',\varphi')\right].\end{aligned} \tag{38}$$

Next, Eqns. (37) and (38) are represented in the following form:

$$L_1^1(\tau,\mu,\varphi)=F\mu_s\left(\rho_{oa}+\rho_{oa}^G\right),\qquad L_2^1(\tau,\mu,\varphi)=F\mu_s\left(\rho_{sk}+\rho_{sk}^S\right). \tag{39}$$

Let the values $\rho_{ik} = \rho_{ik}(\tau)$ be referred to as the radiance factors (RF). Separating the RF of glare ρ_{oa}^{G} and the RF of the solar disk ρ_{sk}^{S} :

$$\rho_{oa}^{G} = \frac{\pi}{\mu_s} R_F(\mu)\delta(\varphi-\pi)\delta(\mu-\mu_s)\exp\left[-(\tau^*+\tau)/\mu_s\right],\ (\mu,\varphi)\in\Omega_+, \tag{40}$$

$$\rho_{sk}^{S} = \frac{\pi}{\mu_s}\,\delta(\varphi)\delta(\mu-\mu_s)\exp\left[-(\tau^*-\tau)/\mu_s\right],\quad (\mu,\varphi)\in\Omega_+. \tag{41}$$

Integration of Eqns. (31) and (32) gives the radiance factor ρ_{oa} of the atmospheric layer situated below the horizontal surface $\tau = \text{const}$, and the radiance factor ρ_{sk} of the sky measured at the level τ:

$$\begin{aligned}\rho_{oa}(\tau) = AT_sT_1 + \frac{\tau^*}{4\mu(\mu_s+B\tau^*)}&\left\{\frac{\tau}{\tau^*}\left[a_1(1-f_1)+g_sd_1\left(1-\left(1-\frac{\mu}{\tau^*}\right)f_1\right)+d_2f_1\frac{\mu}{\tau^*}\right]\right.\\ &\left.+g\left[a_3(1-f_1^*)+d_1\left(1-\left(1-\frac{\mu}{\tau^*}\right)f_1^*\right)+g_sd_2f_1\frac{\mu}{\tau^*}\right]\right\},\end{aligned} \tag{42}$$

$$\rho_{sk}(\tau) = AT_sT_2 + \frac{\tau^*-\tau}{4\mu(\mu_s+B\tau^*)}\left\{a_2(1-f_2)+g_sd_1f_2\frac{\mu}{\tau^*}+d_2\left[1-\left(1-\frac{\mu}{\tau^*}\right)f_2\right]\right\}, \tag{43}$$

where

$$T_1 = 1-\frac{\tau}{\mu}B_\mu(1-f_1)-\frac{\tau}{\tau^*}\left(1-F_\mu-B_\mu\right)+\frac{g}{\mu}\left[f_1^*+\tau^*B_\mu\left(1-f_1^*\right)\right] \tag{44}$$

is the light transmittance of the atmosphere from the ocean surface to the detector,

$$T_2 = \frac{\tau^*-\tau}{\mu}\left[\frac{\mu}{\tau^*}f_2+B_\mu(1-f_2)\right], \tag{45}$$

$$f_1 = f\left(\frac{\tau}{\mu}\right),\ \ f_1^* = f\left(\frac{\tau^*}{\mu}\right),\ \ f_2 = f\left(\frac{\tau^*-\tau}{\mu}\right),\ \ f(x) = 1+\frac{\exp(-x)-1}{x}, \tag{46}$$

$$\left.\begin{aligned}a_1 = p(-\cos\chi_+)+g_sp(\cos\chi_-),\ \ a_2 = p(\cos\chi_+)+g_sp(-\cos\chi_-),\\ a_3 = p(\cos\chi_-)+g_sp(-\cos\chi_+),\\ d_1 = Bm\left[p(\cos\chi_-)+p(-\cos\chi_-)\right],\ \ d_2 = Bm\left[p(\cos\chi_+)+p(-\cos\chi_+)\right],\end{aligned}\right\} \tag{47}$$

$$\left.\begin{aligned}\cos\chi_\pm = \mu\mu_s \pm\sqrt{(1-\mu^2)(1-\mu_s^2)}\cos\varphi,\ \ \bar{p}(\mu,\mu') = (2\pi)^{-1}\int_0^{2\pi}p(\cos\chi)\,d\varphi,\\ B_\mu = 0.5\int_0^1\frac{\bar{p}(-\mu,\mu')}{B\tau^*+\mu'}\mu'd\mu',\quad F_\mu = 0.5\int_0^1\frac{\bar{p}(\mu,\mu')}{B\tau^*+\mu'}\mu'd\mu'.\end{aligned}\right\} \tag{48}$$

In the approximation for the phase function given by Eqn. (8) B_μ, F_μ , T_1 and T_2 can be expressed by the following simple formulas:

$$B_\mu \cong \frac{B}{1+B\tau/\mu},\quad F_\mu \cong \frac{1-B}{1+B\tau/\mu}, \tag{49}$$

$$T_1 \cong \frac{1+g_s(B+0.5)\tau^*/\mu}{1+B\tau^*/\mu},\qquad T_2 = \frac{B(\tau^*-\tau)}{B\tau^*+\mu}. \tag{50}$$

Note that equations for the radiance factors (42) and (43) are valid for any values of τ^*, τ, B, A, μ and μ_s.

3.0 SIMPLIFIED EXPRESSIONS

It is possible to substantially simplify equations for the radiance factors by imposing the following limitations:

$$\tau << \mu_s, \quad f_1 \approx \frac{\tau}{2\mu} << 1, \quad f_2 \approx \frac{\tau^* - \tau}{2\mu} << 1. \tag{51}$$

In this case

$$B_\mu \approx B, \quad F_\mu \approx 1 - B, \quad d_1, d_2 << 1. \tag{52}$$

Retaining values which are linear over small parameters in Eqns. (49) and (50):

$$\rho_{oa} = \frac{\tau\, p(-\cos\chi_+) + \left[\tau\, R_F(\mu_s) + \tau^*\, R_F(\mu)\right] p(\cos\chi_-)}{4\,\mu\,\mu_s} + A T_s T_1\,, \ \ (\mu, \varphi) \in \Omega_+, \tag{53}$$

$$\rho_{sk} = \frac{(\tau^* - \tau)}{4\,\mu\,\mu_s}\left[p(\cos\chi_+) + R_F(\mu_s)\, p(-\cos\chi_-)\right], \ (\mu, \varphi) \in \Omega_+, \tag{54}$$

where

$$T_s \cong 1 - Bm \cong \frac{1}{1 + Bm}, \quad T_1 \cong 1 - \frac{B\tau}{\mu} \cong \frac{1}{1 + B\tau/\mu}. \tag{55}$$

For the case of a homogeneous atmosphere:

$$p(\cos\chi) = \frac{s_R\, p_R(\cos\chi) + s_A\, p_A(\cos\chi)}{s_R + s_A}, \tag{56}$$

where s_R and s_A accept, respectively, the values: τ_R, τ_R^*, $\tau_R^* - \tau_R$ and τ_A, τ_A^*, $\tau_A^* - \tau_A$. The values τ_R, p_R and τ_A, p_A are, respectively, the optical thickness and the phase function of the Rayleigh and aerosol atmospheric components; τ_R^* and τ_A^* are the total optical thicknesses of the Rayleigh and aerosol atmospheres, $\tau = \tau_R + \tau_A$, $\tau^* = \tau_R^* + \tau_A^*$.

For the case of a homogeneous atmosphere both Eqns. (53) and (54) can be rewritten in the much simplified form with the help of Eqn. (56):

$$\begin{aligned}\rho_{oa}(\tau) = {} & \frac{\tau_R p_R(-\cos\chi_+) + \left[\tau_R R_F(\mu_s) + \tau_R^* R_F(\mu)\right] p_R(\cos\chi_-)}{4\mu\,\mu_s} \\ & + \frac{\tau_A p_A(-\cos\chi_+) + \left[\tau_A R_F(\mu_s) + \tau_A^* R_F(\mu)\right] p_A(\cos\chi_-)}{4\mu\,\mu_s} + A T_S T_1, \ \left(\mu, \varphi \in \Omega_+\right)\end{aligned} \tag{57}$$

$$\begin{aligned}\rho_{sk}(\tau) = {} & \frac{\left(\tau_R^* - \tau_R\right)}{4\mu\,\mu_s}\left[p_R(\cos\chi_+) + R_F(\mu_s) p_R(\cos\chi_-)\right] \\ & + \frac{\left(\tau_A^* - \tau_A\right)}{4\mu\,\mu_s}\left[p_A(\cos\chi_+) + R_F(\mu_s) p_A(\cos\chi_-)\right], \ \left(\mu, \varphi \in \Omega_+\right).\end{aligned} \tag{58}$$

At the upper level of atmosphere, *i. e.* at the level of the satellite, where $\tau_R = \tau_R^*$, $\tau_A = \tau_A^*$, Eqn. (57) becomes the well-known expression from an earlier study (Viollier, Tanre and Deschamps, 1980), the accuracy of which is evaluated at 5% with the following restrictions on the angles and the value of atmospheric optical thickness:

$$\theta,\ z_s \le 15^\circ, \quad \tau^* \le 1. \tag{59}$$

From Eqns. (43) and (58) it is possible to obtain expressions for the experimentally measurable radiance indicatrix in the sun's *almucantar*, which is a small circle of equal sun's elevation angle on the hemisphere centered on the zenith axis (McCartney, 1976):

$$\mu_{sk}(\Phi) = \frac{\mu_s^2 \exp(m)}{\pi} \rho_{sk}(\tau)\Big|_{\mu=\mu_0}, \tag{60}$$

where $\Phi = \cos^{-1}\left[\mu_s^2 + \left(1-\mu_s^2\right)\cos\varphi\right]$ is the angular distance from the sun. Assuming that $\cos\chi_+ = \cos\Phi$, $\cos\chi_- = 2\mu_s^2 - \cos\Phi$ and taking into account the restrictions imposed by Eqn.(51) the following equation at sea surface level $(\tau_R,\ \tau_A = 0)$ can be obtained from Eqn.(58):

$$\begin{aligned} \mu_{sk}(\Phi) &= \frac{\tau_R^*}{4\pi}\left[p_R(\cos\Phi) + R_F(\mu_s)\, p_R(\cos\Phi - \cos^2\mu_s)\right] \\ &+ \frac{\tau_A^*}{4\pi}\left[p_A(\cos\Phi) + R_F(\mu_s)\, p_A(\cos\Phi - \cos^2\mu_s)\right]. \end{aligned} \tag{61}$$

which is valid when $\tau_A^* << (\cos\mu_s - \tau_R^*)$. In the general case the more complicated formula for radiance indicatrix obtained from Eqns. (43) and (60) should be used:

$$\mu_{sk}(\Phi) = \frac{\tau^* e^m}{4\pi}\left\{A_2[1 - f(m)] + D_1 f(m) + D_2\left[m - (1+m)\, f(m)\right]\right\} \tag{62}$$

where

$$A_2 = p(\cos\Phi) + R_F(\mu_s)\exp(-m)\, p(\cos\Phi - \cos^2\mu_s), \tag{63}$$

$$D_1 = R_F(\mu_s)\, B e^{-m}\left[p(\cos\Phi - \cos^2\mu_s) + p(\cos^2\mu_s - \cos\Phi)\right], \tag{64}$$

$$D_2 = B\left[p(\cos\Phi) + p(-\cos\Phi)\right], \quad m = \tau^* / \mu_s. \tag{65}$$

Equations (61) or (62)-(65) can be used as an algorithm for the restoration of the atmospheric optical thickness from measurements of the radiance indicatrix in the sun's *almucantar*, or light radiance in the circle of equal sun's elevation angle on the hemisphere.

4.0 VALIDATION

For the more simple case of satellite sounding, results given by the equations derived here have been compared with the results produced by the algorithm based on Gordon and Wang (1994). For viewing angles less than 15 degrees the discrepancies never exceeded 10%. Comparison with results from Monte-Carlo simulations also confirms this estimate.

5.0 CONCLUSION

Fairly simple analytical expressions have been obtained which relate the characteristics of the light field in the atmosphere above the sea with the parameters of the atmosphere and of the underlying surface. Within the constraints of Eqn.(51) and with the assumption that the atmosphere is uniform, expression (42) obtained within the framework of this theory is transformed at the upper boundary of the atmosphere into a well-known formula (Viollier, Tanre and Deschamps, 1980) which has an accuracy of 5% under the constraints of Eqn. (51). For arbitrary angles θ, z_s, and optical thickness τ^*, the accuracy of the expressions obtained here lies in the range of 10-15%.

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